CHAPTER 14

Important Details of Stability Analyses

The reliability of slope stability computations depends on the validity of the soil properties, the slope and subsurface geometry, and the pore water pressures used in the analyses. The reliability of the results is also dependent on several other aspects of the computation procedures. These include the following:

- Method of searching for the critical slip surface and verifying that the most critical slip surface has been located
- Detection and elimination of tensile forces between slices at the top of the slip surface
- Detection and elimination of unreasonably large compressive or tensile forces between slices at the toe of the slip surface
- Evaluation of possible three-dimensional effects

These and several other aspects of slope stability computations are discussed in this chapter.

LOCATION OF CRITICAL SLIP SURFACES

For simple slopes it is possible to estimate the location of the critical slip surface relatively well. For example, for a homogeneous slope composed of dry cohesionless soil with a constant friction angle (linear failure envelope), the critical slip surface is a plane coincident with the face of the slope; the factor of safety is given by the equation for an infinite slope: $F = \tan \phi' / \tan \beta$. For most cases the critical slip surface must be determined by trial and error. Even for a homogeneous slope composed of cohesionless soil, the critical slip surface must be located by trial and error if the Mohr failure envelope is curved or if there is a nonuniform seepage pattern (hydraulic gradients vary) near the face of the slope.

Infinite Slope

The easiest critical slip surface to locate is the one for an infinite slope. For an infinite slope the slip surface is a plane parallel to the face of the slope. Location of the critical slip surface involves finding the depth of the shear plane that produces the minimum factor of safety. For cohesionless soils ($c = 0$ or $c' = 0$) the critical plane is in the layer with the lowest friction angle, unless seepage conditions dictate otherwise. For purely cohesive soils ($\phi = 0$) the critical slip surface will usually be at the depth where the ratio of shear strength to depth, c/z, is a minimum; the critical slip surface will pass to the bottom of any layer that has a constant shear strength. If stability is checked for slip surfaces located at the top and bottom of each soil layer, the critical slip surface will almost always be found.

Circular Slip Surfaces

Locating a critical circle is more complicated than finding the depth of the critical slip surface for an infinite slope. Locating a critical circle requires a systematic search in which the center point of the circle and its radius are varied. Such searches are usually performed using a computer program in which the search is automated. Details of the particular search methods vary from one computer program to another. It is important to understand the particular method used, and to be able to control the search, to ensure that the search is thorough.

Most of the schemes used in computer programs to locate critical circles require that an initial estimate be made of the starting location of the circles used in the search. The initial estimate is usually based on an estimate of the location of the center point and the radius of the critical circle. Depending on the search scheme

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employed either a specific initial trial circle for starting the search is designated or the extent of a grid of center points over which the search will be conducted is specified. Usually, the radius is estimated and designated by specifying one of the following:

- The depth of a line to which circles are tangent
- A point through which circles pass
- The radius of circles

Depending on the specific search scheme used, the initial information may define either the radius for a single initial trial circle or a range in radii to be investigated. The following guidelines can be used to estimate the location of a critical circle for initiating a search.

Center point location. A likely center point for a critical circle can be estimated from what is known about the critical circles for the cases of simple, homogeneous slopes in purely frictional soil (c, $c' = 0$) and purely cohesive ($\phi = 0$) soil. For a cohesionless slope the critical slip surface is a plane coincident with the face of the slope. If a search is performed with circles, the critical ''circle'' is very shallow and has a very large radius. The critical circle degenerates to essentially a plane parallel to the slope surface. The center of the critical circle is located along a line passing through the midpoint of the slope, perpendicular to the face of the slope (Figure 14.1*a*). In contrast, for a purely cohesive slope, the critical circle passes as deep as possible. In this case the center of the critical circle lies along a vertical line passing through the midpoint of the slope as shown in Figure 14.1*b*.[1] In both cases ($c = 0$ and $\phi = 0$) the centers of the critical circles lie on a line passing through the midpoint of the slope, and the line is inclined at an angle, ϕ_d, from vertical (Figure 14.2), where ϕ_d is the ''mobilized'' friction angle (i.e., $\tan \phi_d = \tan \phi / F$). Based on this knowledge, an estimate of the center of the critical circle for other slopes can be made by drawing a line from the midpoint of the slope inclined at an angle, ϕ_d, from vertical (see O–P in Figure 14.2). A sufficient estimate of the factor of safety can usually be made for the purpose of determining the developed friction angle, ϕ_d. A good starting point for searching for the critical circle is a point along the line (O–P) at a distance equal to one or two times the slope height above the crest of the slope (see point C in Figure 14.2). Also, a starting center anywhere in the region between a vertical line and a line drawn perpendicular to the midpoint of the slope (the shaded area in Figure 14.2) would be a reasonable starting center.

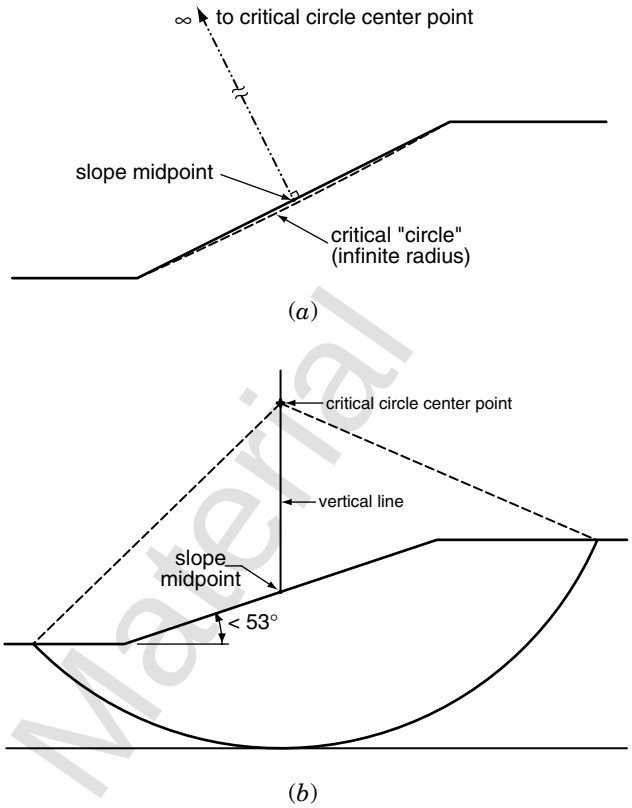


Figure 14.1 Locations of center points for critical circles in (*a*) purely cohesionless (c, $c' = 0$) and (*b*) purely cohesive (ϕ, $\phi' = 0$) slopes.

Radius (depth of circles). When a search is conducted to locate a critical slip surface, it is important to explore circles that pass through all of the different materials in the cross section. If the shear strength of a given stratum is characterized by c = constant, $\phi = 0$, the critical circle will usually, but not always, pass to the bottom of the layer. Circles should be analyzed that pass to the bottom of each stratum that has a constant shear strength. Also, it is usually more effective to start with deep circles and search upward rather than searching downward. By starting the search with circles that are deep, the trial circles should intersect most layers in the cross section, and the circles will be more likely to migrate into the layers that produce the lower factors of safety. Another useful strategy is to search for the critical circle that passes through the toe of the slope and to examine other types of circles (radii and tangent depths), using the critical toe circle as the beginning circle.

Increments for center point coordinates and radius. Most computer programs that employ search schemes using circles vary the coordinates of the cen-

[1] This is true only for slopes flatter than 53°, but covers many of the cases of practical interest.

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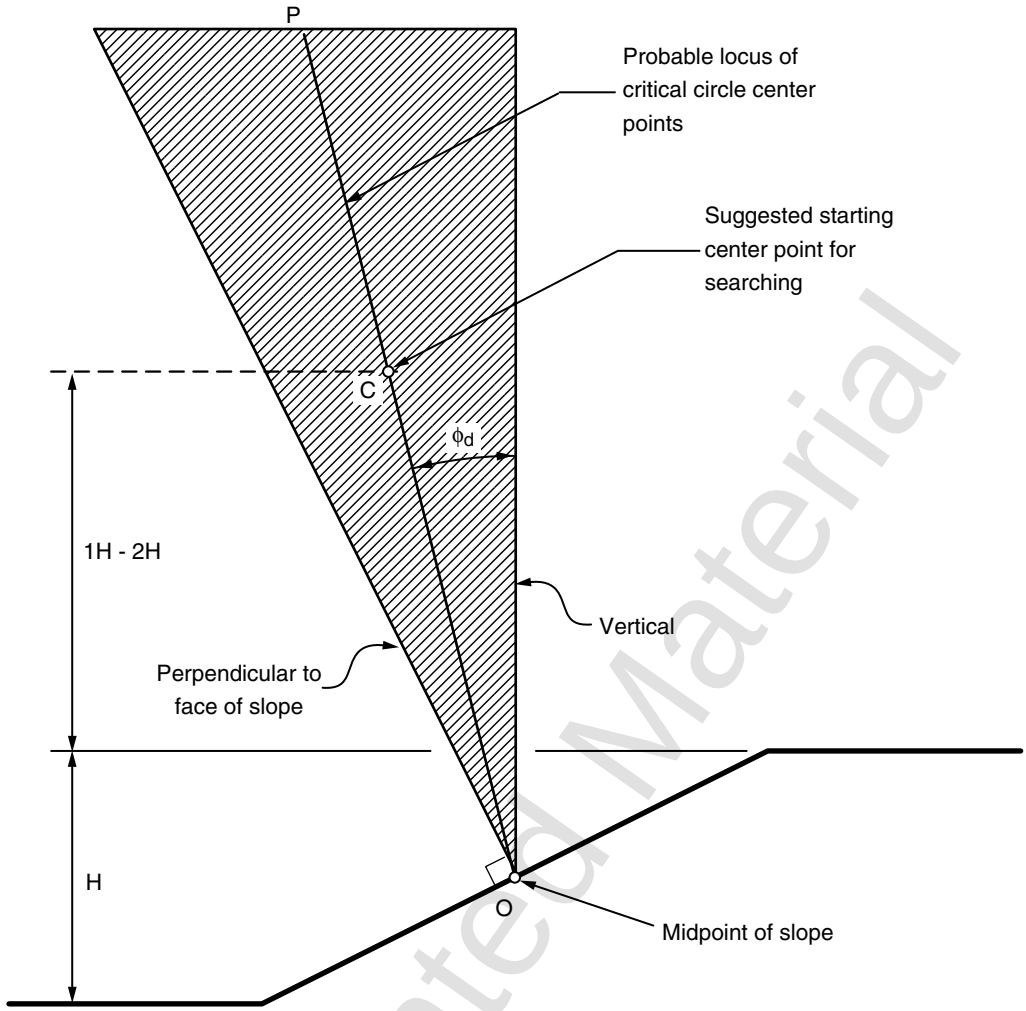


Figure 14.2 Estimation of starting center point to search for critical circles.

ter point systematically and the radius in increments until a minimum factor of safety is found. It is important that the increments used to vary the center point coordinates and the radius be small enough so that important features in the cross section are detected by the search. This requires that the increments be no greater than some fraction of the thickness of the thinnest layer in the cross section. A distance no larger than one-half to one-fourth of the later thickness is effective. Search increments from one-tenth (0.1) to one-hundredth (0.01) of the slope height are suitable if there are no thin layers in the cross section. With the computational speed of current computers a search increment of 1% of the slope height (0.01 H) can be used with little concern for the computation time required.

Multiple minimums. For even simple slopes more than one circle may be found that produces a local minimum for the factor of safety. That is, if the circle is moved slightly in any direction away from the local minimum, the factor of safety will increase. As an illustration, consider the three simple, purely cohesive ($\phi = 0$) slopes shown in Figure 14.3. The slopes shown in this figure are identical except for the thickness of the foundation. The slope shown in Figure 14.3*a* has a 30-ft-thick foundation. The two circles shown in this figure both represent local minimums, with factors of safety of 1.124 for the shallow circle and 1.135 for the deep circle. The shallower circle through the toe of the slope is slightly more critical and thus represents the overall minimum factor of safety. The slope shown in Figure 14.3*b* has a slightly deeper foundation, 46.5 ft deep. This slope also has two circles that produce locally minimum factors of safety, but both circles have the same factor of safety (1.124); there are two critical circles with identical minimum factors of safety. The third slope, shown in Figure 14.3*c*, has the deepest foundation: 60 ft deep. There are again two local minimums for this slope,

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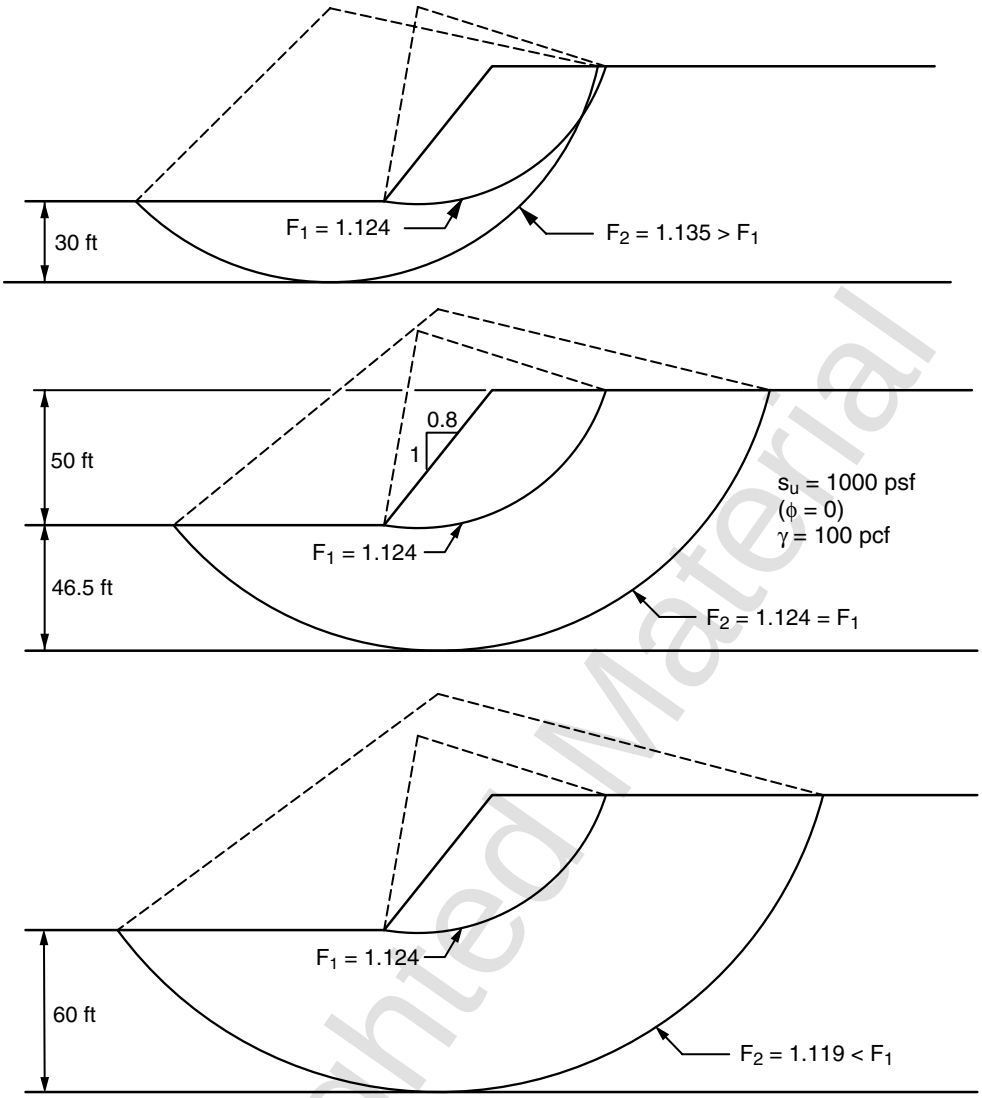


Figure 14.3 Local critical circles with similar or the same factor of safety.

with the shallow circle producing a factor of safety of 1.124 and the deeper circle having a factor of safety of 1.119. In this case the deeper circle represents the more critical of the two local minimums. This example shows that with relatively small changes in the depth of the foundation there may be quite different locations for the critical circle, and in some cases there may even be two different critical circles that have the same minimum factor of safety.

Another example of a slope with two local minimums for the factor of safety is shown in Figure 14.4. For the cohesionless embankment there is a local minimum factor of safety corresponding to a shallow, infinite slope mechanism. The factor of safety for this shallow slip surface is 1.44. Another local minimum occurs for a deeper circle passing to the bottom of the clay foundation. The factor of safety for the deeper circle is 1.40 and is the overall minimum factor of safety for this slope. In cases like the ones illustrated in Figure 14.3 and 14.4, it is important to conduct multiple searches to explore both deep and shallow regions of the slope profile.

When searches for the critical circle are done manually, it is straightforward to explore all regions of the cross section. However, when searches are done using computer programs in which the search is conducted automatically, care is required to ensure that all feasible regions of the slope and foundation have been explored and that the critical circle has been found. Most computer programs with automatic search routines per-

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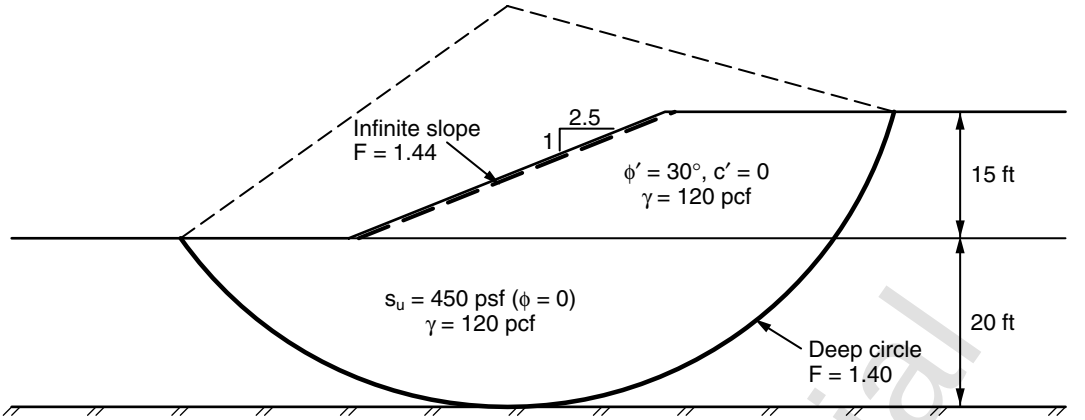


Figure 14.4 Slope with two local critical slip surfaces and minimum factors of safety.

mit the starting point for the search to be designated as input data. Therefore, it is possible to perform a number of independent searches by using different starting locations. It is good practice to perform several searches using different starting locations to ensure that all regions of the cross section are explored and that the most critical slip surface is found.

One scheme for locating a critical circle is to vary the radius for each trial center point until a critical radius, corresponding to the minimum factor of safety for the center point, is found. When the radii are varied for a given center point, multiple local minimums may be found. For example, Figure 14.5 shows the variation in factor of safety with the radius (depth) of a circle whose center point is the center of the most critical circle overall. It can be seen that there are several local minimums for the factor of safety for the designated center point. It is important in such cases to vary the radius between wide limits and in relatively small increments to capture this behavior and to ensure that the minimum factor of safety is found. An appropriate scheme for doing this is to select the minimum and maximum radius (or tangent line elevation) of interest, along with a suitable increment for varying the radius. The increment used to vary the radius should not exceed one-half to one-fourth of the thickness of the *thinnest* layer.

Although varying the radius to find a critical radius for each trial center point is relatively inefficient computationally—many more circles may be analyzed than perhaps are necessary—the scheme can provide useful information. This scheme is particularly useful when contours are to be drawn to show how the factor of safety varies with the location of the center point. The most meaningful contours that can be drawn for a series of selected center points are those for factors of safety corresponding to the critical radius (minimum factor of safety) for each point.

Noncircular Shear Surfaces

Critical noncircular slip surfaces are much more difficult to locate because they involve many more variables than circles. Circles are described by three values: the x and y coordinates of the center, and the radius. Noncircular slip surfaces are described by two values (x and y) for each point on the slip surface.[2] Depending on the number of points required to characterize the noncircular slip surface, the number of variabes in the search can be quite large.

Two basically different approaches have been used to vary the positions of noncircular slip surfaces and find the slip surface with the minimum factor of safety. One approach employs a systematic shifting of the slip surface, with an appropriate minimization or optimization scheme. Implementations of this method vary from sophisticated schemes employing linear programming methods to much simpler, direct numerical techniques (e.g., Baker, 1980; Celestino and Duncan, 1981; Arai and Tagyo, 1985; Nguyen, 1985; Li and White, 1987; Chen and Shao, 1988).

The second approach for finding a critical noncircular slip surface employs a random process to select trial slip surfaces. This scheme generally proceeds until a specified number of slip surfaces have been explored (e.g., Boutrop and Lovell, 1980; Siegel et al., 1981). A random number generator is used to establish the location of the random slip surfaces.

[2] The exception to this is at the ends of the slip surface, where the points are constrained additionally to be either on the surface of the slope or at the bottom of a crack profile.

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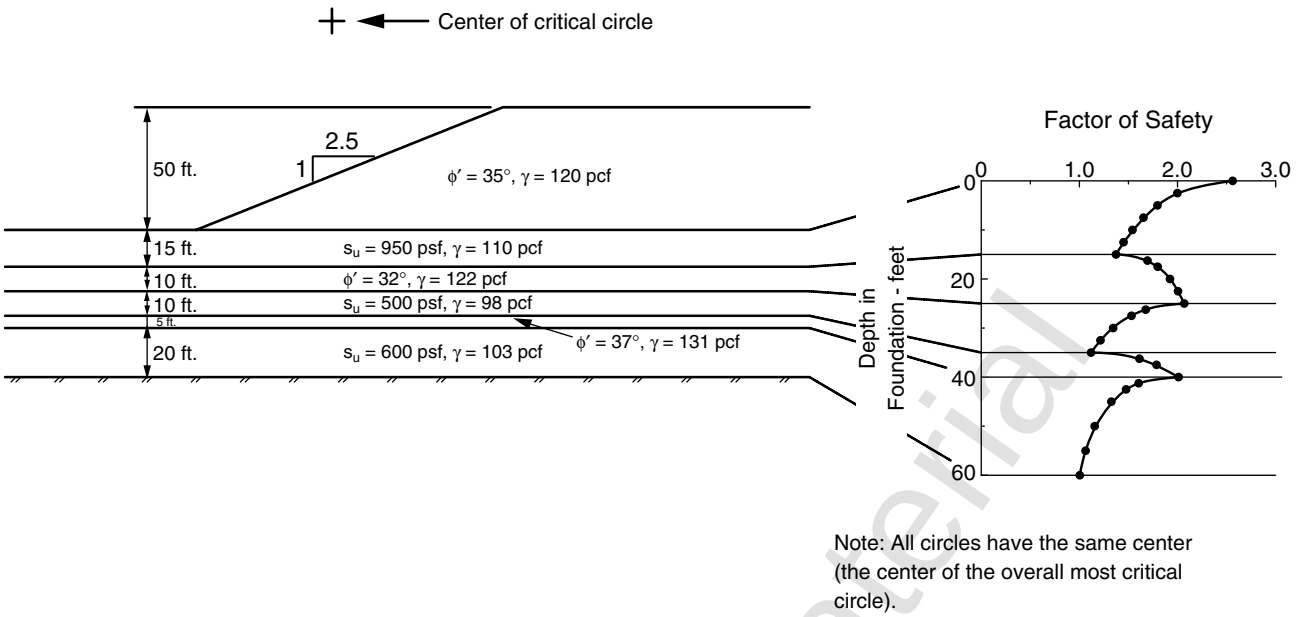


Figure 14.5 Variation of the factor of safety with depth (radius) of circles for an embankment on a stratified soil foundation.

Regardless of the scheme used to find a critical noncircular slip surface, the potential location of the critical slip surface must usually be estimated either as a starting point for the search or to establish limits on the search. A good estimate for the starting location of the noncircular slip surface is the location of the critical circular slip surface. Locating the critical circle and then using several (4 to 10) points along the critical circle to define an initial trial noncircular slip surface works well in many cases. The locations of the initial points are varied in accordance with whatever search scheme is being used to find a more critical noncircular slip surface. Additional points may be added to the noncircular slip surface as the search progresses. This scheme works well in most cases, particularly where there are not very thin, weaker zones of soil. Alternatively, if distinctly weaker zones exist, a slip surface passing through the weaker zones may be selected as a starting point.

When a particularly thin, weak layer is likely to control stability, it is usually better to start with a noncircular slip surface that has a portion of the slip surface following along the weak layer. Such a surface may enter or exit the slope through stronger overlying material. When it does, the inclination of the slip surface at the crest and at the toe of the slope should be chosen to conform to the inclinations of active and passive shear planes, respectively. Active and passive zones are discussed later in this chapter. Near the crest of the slope the slip surface will usually be chosen to enter at an angle of 45 to 65° from the horizontal. Near the toe of the slope the slip surface will usually be chosen to exit at an angle of 25 to 45° from the horizontal.

As with circles, it is important with noncircular slip surfaces to choose several starting locations for the slip surface and perform searches beginning with each. The initial slip surfaces should pass through the various zones of soil that are expected to influence stability. It is also important that sufficiently small increments be used to vary the position of points on the slip surface so that the effects of thin layers are evaluated accurately. Increments no greater than one-half the thickness of the thinnest significant stratum and as small as 1% of the slope height (0.01 horizontal) are appropriate. In most cases it should be anticipated that more than one local minimum exists.

Importance of Cross-Section Details

In embankment dams with complex cross sections, there may be numerous zones of different materials that result in at least several local minimum factors of safety. It is important that each potential slip surface and mechanism of sliding be explored. It is also important to model the cross section in as much detail as possible, because these details may have an effect on the stability. For example, the slope protection (riprap) and associated filters on the upstream face of an earth dam can have a large effect on the stability and safety against shallow sliding due to drawdown. Omission of

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the shallow layers of slope protection in the geometry used for an analysis of sudden drawdown may result in the factor of safety being significantly underestimated, and unnecessary conservatism in the design.

To illustrate the importance of cross-section details, consider the embankment dam shown in Figure 14.6. The embankment is to be subjected to a rapid drawdown of 19 ft, and slope stability computations were performed using the U.S. Army Corps of Engineers (1970) procedure for rapid drawdown. Computations were performed with and without the riprap and filter zones included in the cross section used for analysis. When the filter and riprap were excluded, the material was assumed to be the same as the embankment material. Factors of safety are summarized in Figure 14.6. The factor of safety with the riprap and filter zones included in the analyses was 1.09. The factor of safety dropped to 0.84 when the riprap was excluded—a decrease in factor of safety of approximately 23%. Considering that the U.S. Army Corps of Engineers (1970) recommended factor of safety for this case is 1.0, the effect of omitting the riprap and filters from the analysis was to change the factor of safety from acceptable to unacceptable.

Recapitulation

- An initial estimate of the critical circle can be made from what is known about the critical circles for homogeneous slopes (Figure 14.2).
- Searches should be conducted beginning at several starting points to fully explore the soil profile and detect multiple minimums.
- Searches are usually more successful when the search is started deep rather than shallow.
- Increments of distance used to move the center and change the radius of circles in a search should not exceed one-half the thickness of the thinnest stratum of interest. Increments are typically chosen to be 0.01 to 0.1 times the slope height. The speed of current computers allows use of small increments with no practical penalty.
- Good starting locations for a search with noncircular slip surfaces can be estimated by starting from the critical circle, or by examining the slope cross section to identify layers of weakness.
- The smallest increments used to move the points when searching with noncircular slip surfaces should not exceed one-half the thickness of the thinnest stratum of interest and are typically chosen to be 0.01 to 0.1 times the slope height.
- All cross section details that may influence the position of the critical slip surface and factor of safety should be included in the geometry used for the slope stability analysis.

EXAMINATION OF NONCRITICAL SLIP SURFACES

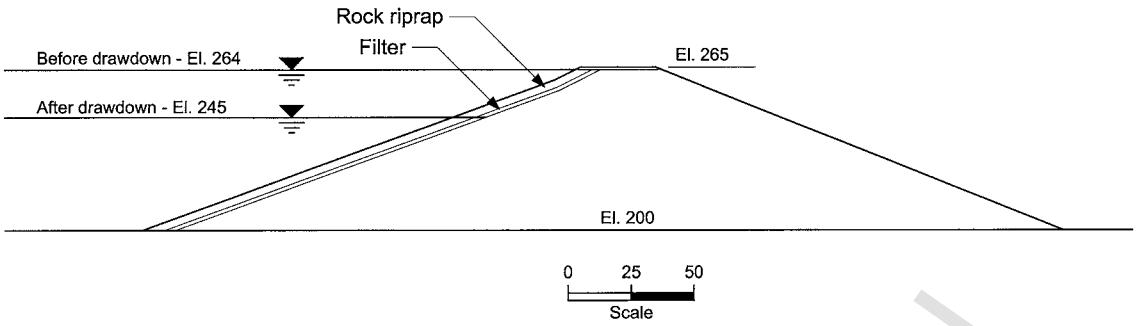
In some cases the slip surface with the minimum factor of safety may not be the slip surface of greatest interest. For example, the minimum factor of safety for the embankment shown in Figure 14.7 is 1.15. The minimum factor of safety corresponds to an infinite slope failure in the cohesionless fill. Also shown in Figure 14.7 is a deeper circle that has a locally minimum factor of safety. This deeper circle has a factor of safety of 1.21, which is higher than the factor of safety for the shallower, infinite slope slip surface. However, if sliding occurred along the deep circle, it would have a far more severe consequence. Failure along the shallow surface in the embankment would consist simply of raveling of material down the slope and might, at the most, represent a maintenance problem. In contrast, failure along the deeper surface might require reconstruction of the embankment. Consequently, the deeper surface and the associated factor of safety of 1.21 would be considered unacceptable, while a factor of safety of 1.15 for the shallower, most critical slip surface might be acceptable.

More than one mechanism of failure and associated factor of safety must sometimes be examined for design. It is good practice to locate as many locally critical slip surfaces as possible. In some cases it is also useful to examine slip surfaces that are important but do not represent local minimum factors of safety. A good example of cases where noncritical slip surfaces need to be studied is older mine tailings disposal dams. Many older tailing dams consist predominately of cohesionless materials where the critical slip surface is a shallow ''skin'' slide. The consequences of shallow sliding may be minor, but deeper sliding must be avoided. To address this concern, stability computations can be performed using slip surfaces that extend to various depths. The variation of the factor of safety with the assumed depth of slide is then examined and a judgment is made regarding the depth of sliding that can be tolerated. Minimum factor of safety requirements are established for slip surfaces that extend deeper than the tolerable limit. To investigate deeper slip surfaces that do not necessarily represent minimum factors of safety, several techniques can be used to define the slip surfaces that will be analyzed. Three techniques for doing this are illustrated in Figure 14.8. They consist of:

1. Forcing trial slip surfaces to be tangent to a line that is parallel to and below the surface of the slope (Figure 14.8*a*)
2. Forcing trial slip surfaces to pass through a designated point that is located at some depth below the surface (Figure 14.8*b*)

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Description	Factor of Safety
Filter and riprap included in analyses	1.09
No filter and riprap - homogeneous embankment	0.84

Figure 14.6 Effect of included cross-section detail on computed factor of safety for rapid drawdown.

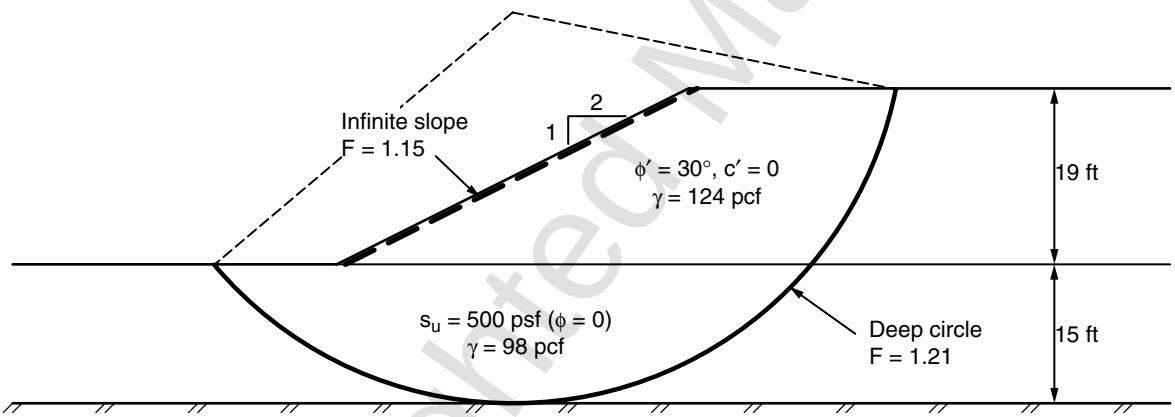


Figure 14.7 Slope with shallow critical slip surface and deeper, locally critical circle.

3. Requiring that the soil mass above the slip surface have some minimum weight (Figure 14.8*c*)

Many computer programs provide some capability for limiting the extent of a search for a critical slip surface in one or more of the ways described above.

Earth dams represent another case where there are slip surfaces besides the slip surface producing the minimum factor of safety that are of interest. Stability analyses must be performed for both the upstream and downstream slopes of dams. One slope (upstream or downstream) typically has a lower factor of safety than the other. In searching for a critical circle on one face of the dam it is possible that the search will ''hop'' to the other face and abandon searching further for the critical slip surface beneath the slope face of interest. If this occurs, it is necessary to employ some means to restrict the search to a specific slope face. Some computer programs automatically restrict the search to the slope face where the search was started; other programs may require that additional constraints be used.

Recapitulation

- The slip surface with the minimum factor of safety is not always the one of greatest interest.
- It is often desirable to examine slip surfaces and factors of safety that are not minimums, especially when the consequences of other mechanisms of failure are important.

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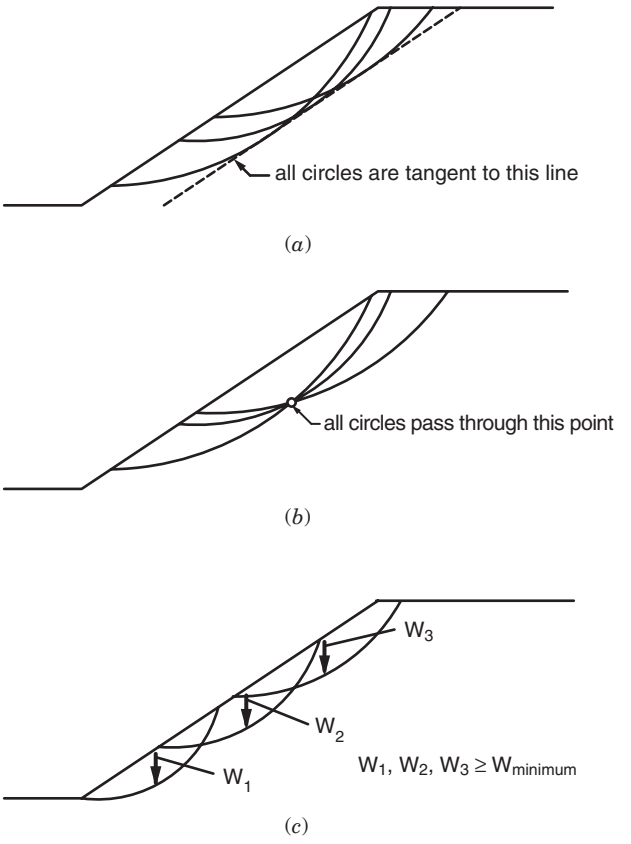


Figure 14.8 Artificial constraints imposed to examine noncritical slip surfaces of interest: (*a*) all circles tangent to a line; (*b*) all circles pass through a fixed point; (*c*) all circles have minimum weight.

TENSION IN THE ACTIVE ZONE

When there are cohesive soils in the upper portion of a slope, slope stability calculations usually will reveal tension at the interfaces between slices as well as on the bottom of the slices. The existence of tension may be of concern for two reasons:

1. Most soils do not have significant tensile strength and thus cannot withstand tension. Calculated tensile stresses are unrealistic and inappropriate.
2. When significant tension develops it can cause numerical problems in the slope stability calculations.

For both reasons it is desirable to eliminate the tensile forces from the analyses.

Rankine Active Earth Pressures

The tension at the top of the slope that occurs in slope stability analyses is analogous to the tensile stresses that are computed from Rankine active earth pressure theory. In fact, the mechanisms that produce tension in slope stability analyses and active earth pressure calculations are the same. For total stresses the Rankine active earth pressure beneath a horizontal ground surface, σ_h, is given by

$$\sigma_h = \gamma z \tan^2\left(45 - \frac{\phi}{2}\right) - 2c \tan\left(45 - \frac{\phi}{2}\right) \quad (14.1)$$

where z is the depth below the ground surface (Figure 14.9). At the ground surface, where $z = 0$, the expression for the horizontal stress is

$$\sigma_h = -2c \tan\left(45 - \frac{\phi}{2}\right) \quad (14.2)$$

which indicates that stresses are negative (tensile) if c is greater than zero. Negative stresses extend to a depth, z_t, given by

$$z_t = \frac{2c}{\gamma \tan(45 - \phi/2)} \quad (14.3)$$

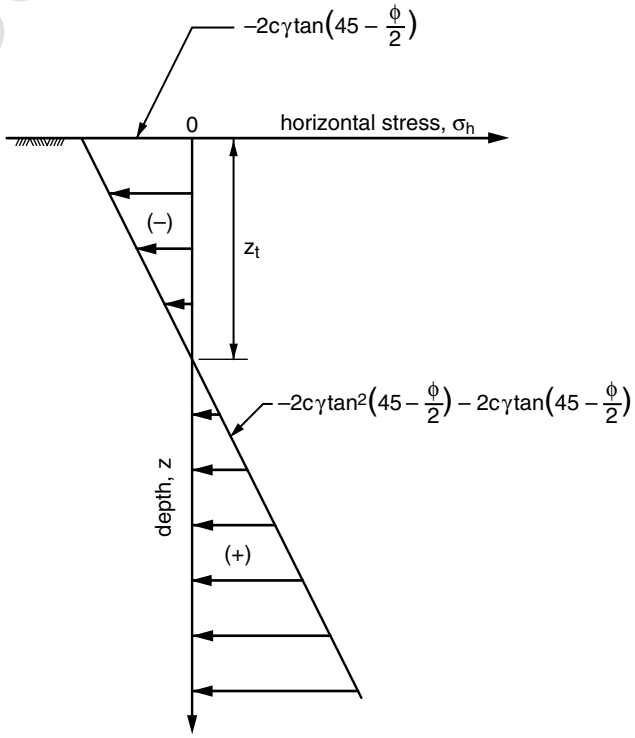


Figure 14.9 Active Rankine horizontal earth pressures beneath a horizontal ground surface.

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The active earth pressures increase linearly from the negative value given by Eq. (14.2) to zero at the depth, z_t. Below the depth z_t the active earth pressures are positive and continue to increase linearly with depth.

Rankine's theory for active earth pressures and the limit equilibrium slope stability analysis procedures described in Chapter 6 are similar for soil near the crest of a slope. Both employ equations of static equilibrium to compute the stresses on vertical planes; in the case of slope stability analyses the interslice forces represent the stresses on vertical planes.

There are also some differences between Rankine earth pressures and limit equilibrium slope analysis procedures. For Rankine earth pressures the forces on vertical planes are parallel to the ground surface; while for limit equilibrium slope stability analyses, various assumptions are made for the direction of forces, depending on the particular procedure of slices used. Also, for Rankine earth pressures the shear strength is assumed to be fully developed ($F = 1$), while for slope stability the shear strength is generally not fully developed (i.e., $F > 1$).

Despite the differences between slope stability and earth pressure calculations, the fundamental cause of tension is the same: Tension is due to some or all of the shear strength of the soil being mobilized, when the soil is assumed to have cohesion. As shown in Figure 14.10, if a Mohr–Coulomb failure envelope with cohesion is assumed, tensile strength is implied. The Mohr circle shown in Figure 14.10 has a major principal stress that is positive while the minor principal stress is negative. Although cohesion may be appropriate for describing the position of a failure envelope for positive normal stresses (σ, $\sigma' > 0$), the same envelope is generally not appropriate for negative (tensile) normal stresses. To achieve more realistic results, it is necessary to ignore the implied tensile strength.

Tension can appear in the results of limit equilibrium slope stability computations in three different ways:

1. The interslice forces become negative.
2. The total or effective normal forces on the bases of slices become negative.
3. The line of thrust moves outside the slice.

Theoretically, at the point where the compressive and tensile stresses are equal on the boundary between slices the line of thrust must lie an infinite distance away from the slope (Figure 14.11). At the point where the compressive and tensile stresses produce equal resultant forces, there is a finite moment but no resultant force to produce the moment. Thus, an infinite moment arm (line of thrust) is required.

Tension may appear in one or more of the three forms listed above. For the Simplified Bishop procedure, interslice forces are not computed, and thus tension appears only in the form of negative normal forces on the bottom of slices. For force equilibrium procedures such as the Modified Swedish and Simplified Janbu procedures, the interslice forces are computed, but not their locations. Thus, in these procedures tension may appear in the form of negative interslice forces and negative normal forces on the base of slices. Finally, for procedures such as Spencer's that consider complete equilibrium, tension may appear in any of the three forms listed above.

Eliminating Tension

Two different techniques can be used to eliminate the effects of tension from the results of slope stability calculations:

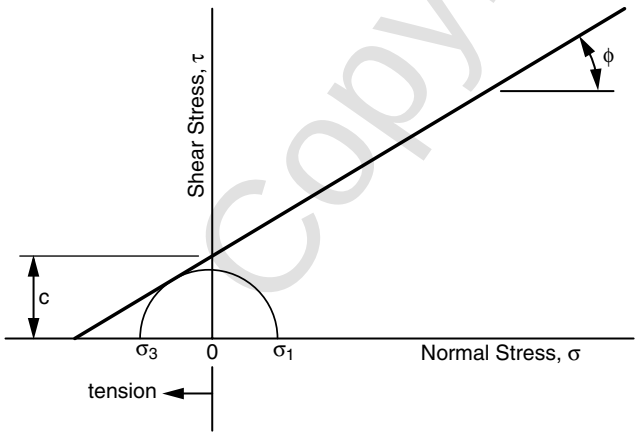


Figure 14.10 Mohr's circle with tensile stresses for a cohesive soil.

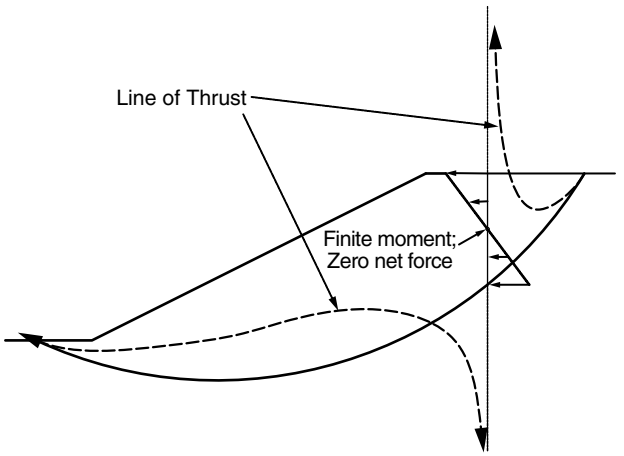


Figure 14.11 Lateral compressive and tensile stresses on an interslice boundary that produce a line of thrust at infinity.

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1. A tension crack can be used in the slope stability calculations.
2. The Mohr failure envelope can be adjusted so that there is no shear strength when the normal stress becomes negative.

Tension crack. A tension crack is introduced into slope stability calculations by terminating the slip surface at the edge of a slice at an appropriate depth below the ground surface (Figure 14.12). The depth of tension can be estimated from Eq. (14.3) using the developed shear strength parameters c_d and ϕ_d. The estimated depth for the vertical crack is given by

$$d_{\text{crack}} = \frac{2c_d}{\gamma z \tan(45 - \phi_d/2)} \tag{14.4}$$

Although c_d and ϕ_d depend on the factor of safety, values can usually be estimated with sufficient accuracy for calculating the depth of crack before the factor of safety is calculated. If necessary, the values for c_d and ϕ_d can be adjusted and the calculations repeated with a revised crack depth once an initial factor of safety is calculated with the estimated crack depth.

In general, when a vertical crack is introduced, the crack should not extend significantly beyond the depth of tension. If the crack depth is overestimated, compressive forces will be eliminated and the factor of safety will be overestimated. In many cases a tension crack has only a minor effect on the computed factor of safety. However, one reason for introducing a tension crack is to eliminate numerical stability problems and inappropriate negative stresses. Thus, even though a tension crack may not have a significant effect on the computed factor of safety, it is a good practice to introduce a crack when there are cohesive soils present along the upper portion of the slip surface.

In most cases only an approximate estimate of the depth of tension crack is needed and can be made based on considerations of active earth pressures as described above. However, if the effect of tension is unclear, a series of stability computations can be performed in which a vertical crack is introduced and the depth is varied. Typically, these analyses will show that the factor of safety first decreases as the crack depth is increased and tension is eliminated and then increases as the crack depth becomes greater and compressive stresses are eliminated as well (Figure 14.13). Such analyses are useful where there are questions regarding the effects of tension and the appropriate crack depth.

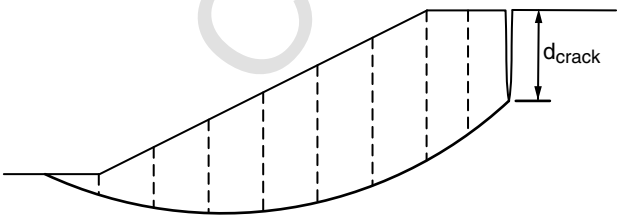


Figure 14.12 Slope and slip surface with a tension crack to eliminate tensile stresses.

Zero tensile strength envelope. Instead of introducing a tension crack, the shear strength envelope can be adjusted so that the shear strength is zero when there is tension. This can be accomplished using a nonlinear Mohr failure envelope like either of the ones shown by the heavy lines in Figure 14.14. Trial-and-error procedures are then required to determine the appropriate shear strength because the shear strength parameters (c, c' and ϕ, ϕ') depend on the normal stress (σ, σ'). If the Mohr failure envelope has abrupt changes in slope, numerical instability and convergence problems can occur in these trial-and-error procedures. That is, the shear strength may oscillate between zero and some finite value of shear strength on successive iterations. Thus, although use of a nonlinear failure envelope may be a good approach from a fundamental viewpoint, appropriate envelopes are difficult to determine, and their use can cause additional numerical problems. Use of a tension crack as described earlier

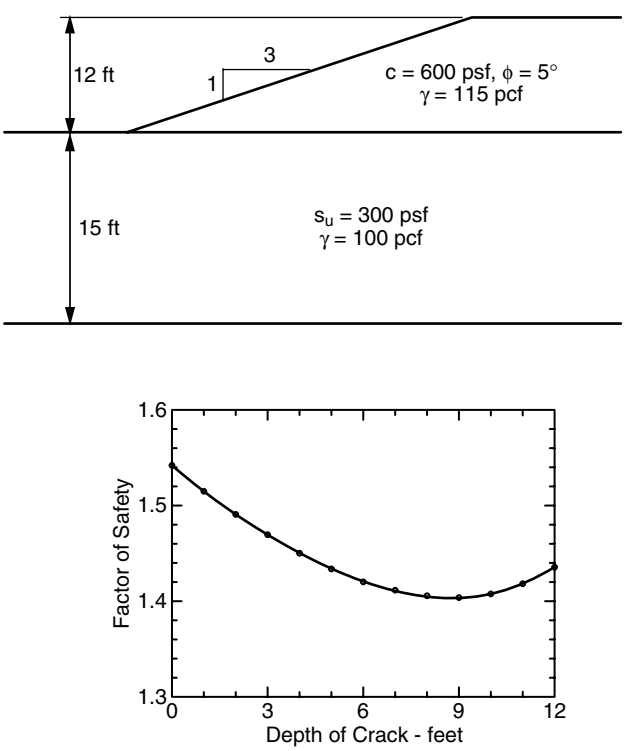


Figure 14.13 Variation in factor of safety with the assumed depth of crack.

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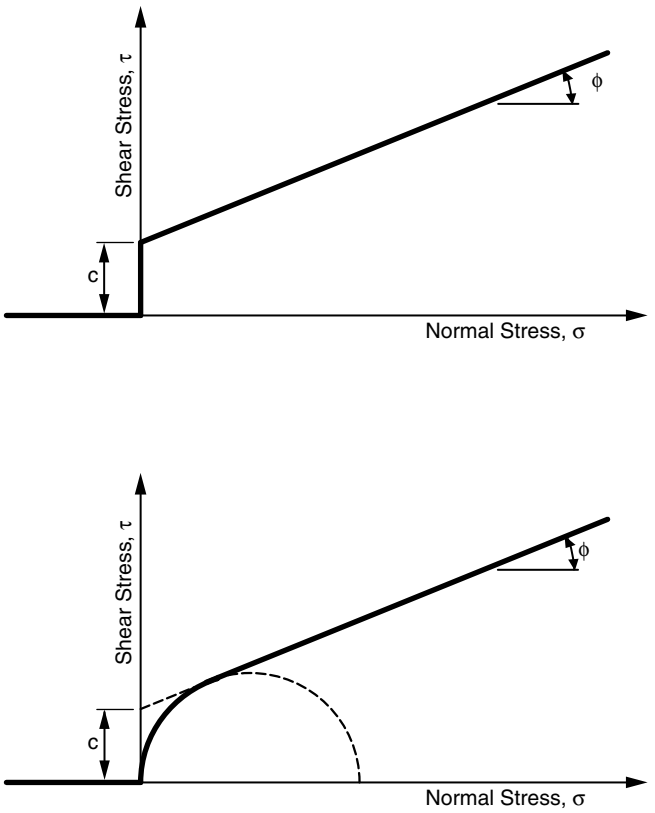


Figure 14.14 Nonlinear Mohr failure envelopes (heavy lines) used to prevent tension in cohesive soils.

is usually a more practical alternative for eliminating tension.

Replacing Cracked Embankments by Surface Loads

In some cases, particularly where relatively strong cohesive embankments rest on much weaker foundations, the crack depth calculated from Eq. (14.4) may exceed the height of the embankment. If a crack depth equal to the height of the embankment is assumed, the embankment strength plays no role in the computed factor of safety. The factor of safety calculated with a tension crack extending through the full height of the embankment is identical to the factor of safety that is calculated assuming a vertical surcharge, represented by a distribution of vertical stress such as the one shown in Figure 14.15*a*.

When an embankment is treated as a vertical surcharge, the shear strength of the embankment has no effect on the computed factor of safety. This, however, is not equivalent to the case where an embankment is assumed to have no, or very little, shear strength. If an embankment or fill has negligible strength, there will be a significant horizontal thrust that is not represented by a vertical surcharge (see P_f in Figure 14.15*b*). It is

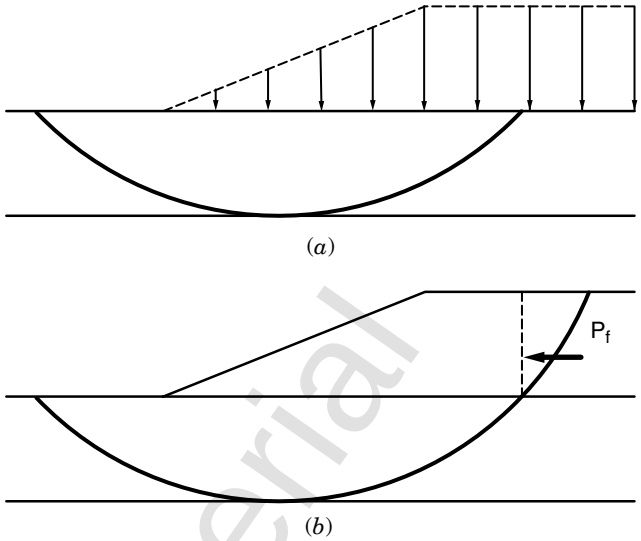


Figure 14.15 Load representations for embankments where the shear strength is neglected: (*a*) vertical surcharge; (*b*) lateral thrust produced by weak fill.

appropriate in the latter instances to model the fill as a low strength material by assigning small or zero values for the shear strength parameters (c and ϕ). If the embankment is assigned a small or zero shear strength, the appropriate lateral thrust will be reflected in the limit equilibrium calculations.

Recapitulation

- Tension can cause numerical stability problems in the solution for the factor of safety.
- Tension can imply tensile strength that does not exist and will result in a calculated factor of safety that is too high.
- Introducing a tension crack can eliminate the adverse effects of tension.
- The depth for a tension crack can be estimated from simple equations derived from earth pressure theory.
- Neglecting the strength of an embankment by introducing a tension crack or treating the embankment as a vertical surcharge is appropriate when the embankment is very strong, but is not correct when the embankment is very weak.

INAPPROPRIATE FORCES IN THE PASSIVE ZONE

In the preceding section problems that can develop near the crest of the slope were discussed. Other prob-

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lems can develop near the toe of the slope. As with the problems near the crest of the slope, earth pressure theories are helpful in understanding the stresses and problems that can develop near the toe of a slope and slip surface. The region near the toe of a slip surface corresponds to a zone of passive earth pressures. Problems in the form of very large compressive or even tensile stresses can develop near the toe of the slope.

Cause of Problems

Problems develop near the toe of a slope when the direction of the resultant force, R, on the base of the last slice is very close to the direction of the interslice force, Z. In this case the resultant force on the base of the slice and the interslice force become either very large or negative. This is illustrated for a typical slice in Figure 14.16. It is assumed in this case that the soil is cohesionless. The resultant force (R) due to the normal stress and the mobilized shear strength acts at the angle, ϕ_d, from a line perpendicular to the base of the slice. The slice shown in Figure 14.16 has a base slope angle (α) of $-55°$ and the mobilized friction angle is 25°. The resultant interslice force inclination is 10°. The interslice force and the resultant force on the base of the slice both have the same line of action. There is no force perpendicular to these two forces (R and Z) to balance the weight of the slice. Consequently, mathematically the interslice force and the force on the base of the slice become infinite. This condition is reflected by the following term that appears in the denominator of the equations for the interslice force resultant, Q, presented in Chapter 6 for Spencer's procedure:

$$\cos(\alpha - \theta) + \frac{\sin(\alpha - \theta) \tan \phi'}{F} \qquad (14.5)$$

This quantity appears in the denominator of one of the terms for the interslice force resultant (Q). Substituting values, $\alpha = -55°$, $\tan \phi'/F = \tan \phi_d = \tan 25°$, and $\theta = 10°$ into (14.5) produces a value of zero. As a result, the equations used to compute the factor of safety by Spencer's procedure contain zero in the denominator for one or more slices, and the forces Q and N become infinite. Whitman and Bailey (1967) noted that a similar problem occurs in the Simplified Bishop procedure when the following term, designated as m_α, becomes small:

$$m_\alpha = \cos \alpha + \frac{\sin \alpha \tan \phi'}{F} \qquad (14.6)$$

This term is identical to the term in Eq. (14.5) from Spencer's procedure when the interslice force inclination (θ) is set to zero, as is done in the Simplified Bishop procedure. The problem that is illustrated in Figure 14.16 and is shown by Eqs. (14.5) and (14.6) exists with all limit equilibrium procedures of slices except the Ordinary Method of Slices, which ignores the forces on vertical slice boundaries.

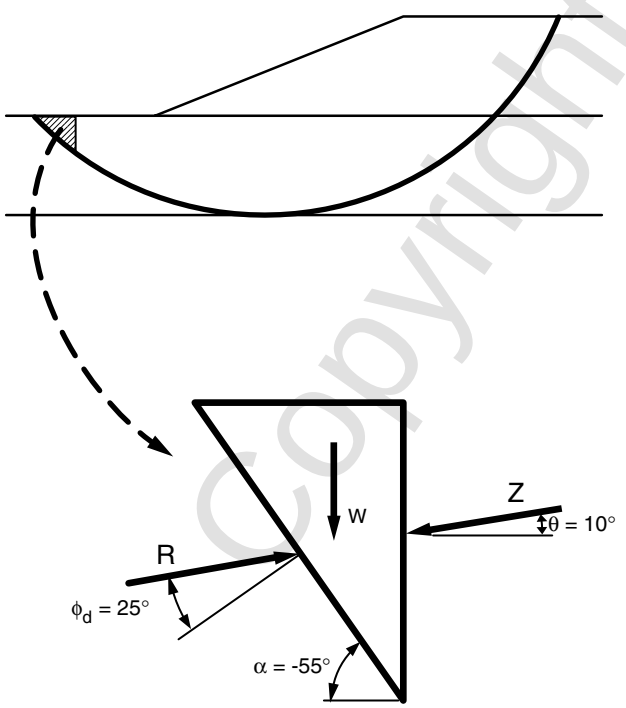


Figure 14.16 Forces leading to infinite values at the toe of the slip surface.

Eliminating the Problem

When the conditions described in the preceding paragraph occur, any of the following may happen:

1. The trial-and-error solution for the factor of safety may not converge—the solution may "blow up."
2. Forces may become extremely large and produce very high shear strengths in frictional materials.
3. Forces may become negative (tensile).

Mathematically, negative stresses in frictional materials will produce negative shear strengths! If this occurs, the factor of safety may be much smaller than reasonable. In this case, if an automatic search is being performed to locate a critical slip surface, the search may suddenly pursue an unrealistic minimum as a solution. The solution may be correct from a mathematical perspective, being a valid root to the simultaneous equations and representing a minimum value for F; however, such a solution is clearly not realistic phys-

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ically and should be rejected. Whitman and Bailey (1967) suggested that when the Simplified Bishop procedure is being used and the value of m_α expressed by Eq. (14.6) becomes less than 0.2, alternative solutions should be explored.

The problem of negative stresses near the toe of the slip surface can be eliminated in at least four different ways. These are described below.

Change the slip surface inclination. Large or negative forces at the toe of the slip surface occur because the inclination of the interslice force and slip surface are different from those corresponding to critical conditions. In other words, the inclinations of the interslice force and slip surface are significantly different from the inclinations corresponding to the minimum passive earth pressure. The relationship between the slip surface inclination and the interslice force inclination for minimum passive earth pressures has been presented by Jumikis (1962) and is illustrated in Figure 14.17. If the assumption pertaining to the inclinations of the interslice forces in a solution are reasonable, as they usually are, the most realistic remedy for inappropriate forces in the passive zone is to change the inclination of the slip surface near the toe of the slope. The inclination of the slip surface that should be used for a given interslice force inclination, and developed shear strength (ϕ_d) can be estimated from Figure 14.17.

Changing the slip surface inclination is the most appropriate way to eliminate inappropriate stresses in the passive zone. However, if calculations are being performed using circular slip surfaces, this remedy cannot be used because the overall geometry of the circle determines the orientation of the toe of the slip surface. In such cases one of the three remaining alternatives described below may be adopted.

Compute strength independently. Ladd (personal communication) has suggested one alternative remedy for tension in the passive zone. He suggested that the shear strength in the passive zone can be estimated independent of the slope stability calculations based on the stresses calculated using Rankine passive earth pressure theory. If the overlying ground surface is horizontal and the soil is cohesionless, the vertical and horizontal stresses, σ_h and σ_v, respectively, are principal stresses that can be calculated as follows:

$$\sigma_v = \sigma_3 = \gamma z \tag{14.7}$$

$$\sigma_h = \sigma_1 = \gamma z \tan^2\left(45 + \frac{\phi'}{2}\right) \tag{14.8}$$

The corresponding shear strength on the slip surface is then determined from the following:

$$\tau_{ff} = \frac{\sigma_1 - \sigma_3}{2} \cos \phi' = \frac{1}{2} \gamma z \left[\tan^2\left(45 + \frac{\phi'}{2}\right) - 1\right] \tag{14.9}$$

where τ_{ff} is the shear stress on the failure plane at failure.

The shear strength, $s = \tau_{ff}$, can be calculated from Eq. (14.9) and entered into the slope stability calculations as a cohesion ($s = c = \tau_{ff}$) that increases linearly with depth and with $\phi = 0$. This approach is simple and seems to work reasonably well. If the zone where the problem occurs does not have a major impact on the stability of the slope, Ladd's simple assumption is adequate.

Use the Ordinary Method of Slices. Very large or negative normal stresses at the toe of the slope do not occur in the Ordinary Method of Slices. The normal force on the base of the slice varies from a value equal to the weight of the slice (when the base is horizontal) to zero (as the base of the slice approaches vertical). Thus, if problems occur in the passive zone with other limit equilibrium procedures, the Ordinary Method of Slices can be used.

Change the side force inclination. The final remedy for eliminating tension near the toe of the slip surface is to change the inclination of the interslice forces in the area where the problem occurs. Figure 14.17 can be used to determine an appropriate interslice force inclination that is consistent with the slip surface inclination in the passive shear zone. Depending on the developed friction angle (ϕ_d) and slip surface inclination (α), an appropriate interslice force inclination (θ) can be determined from Figure 14.17. The inclination can be used directly as the assumed interslice force inclination in force equilibrium slope stability calculations. However, this can only be done with force equilibrium procedures. The interslice force inclination cannot be changed directly in other limit equilibrium procedures, although the inclination can be changed indirectly in the Morgenstern and Price and Chen and Morgenstern procedures through the assumed values for $f(x)$ and $g(x)$.

Discussion. Slope stability calculations in which critical noncircular slip surfaces have been located show that the critical slip surface exits the toe of the slope at angles very similar to what would be expected based on passive earth pressure theory (i.e., the inclinations agree well with those shown in Figure 14.17). Thus, it is generally sufficient to initiate a search with a reasonable starting angle (45° or less, depending on the shear strength properties of the soil), and the critical slip surface is then usually found without problems. It is also helpful to place constraints on the

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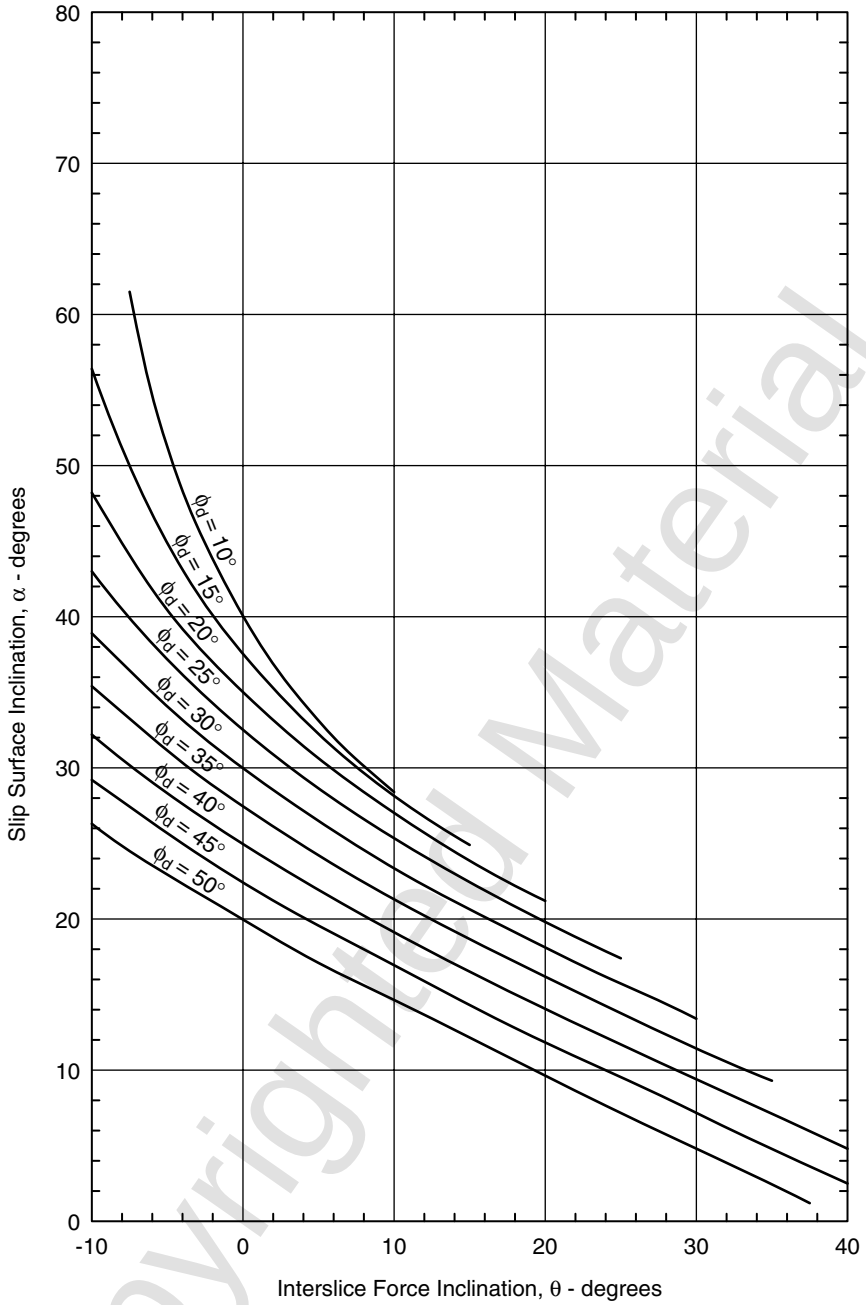


Figure 14.17 Combinations of slip surface inclination and interslice (earth pressure) force inclination for minimum passive earth pressures.

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inclination of the slip surface in an automatic search to avoid very steep slip surfaces being considered where unreasonable solutions may exist due to negative stresses at the toe of the slope. Many computer programs that search for critical noncircular slip surfaces contain provisions for limiting the steepness of the slip surfaces where they exit the slope.

Use of the appropriate passive inclination for the slip surface appears to be the most realistic solution to problems at the toe of the slope and probably conforms most closely with conditions in the field. The only instances where one of the other techniques described needs to be employed is when circular slip surfaces are being used. In these cases the second and third op-

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tions—calculation of the strength independently, and using the Ordinary Method of Slices—usually work well.

Recapitulation

- Very large positive or negative forces may develop on the sides and bottom of slices when the slip surface is steep near the toe of the slope, and the forces act in directions that are significantly different from the directions corresponding to minimum passive earth pressures.
- The problem of inappropriate stresses at the toe of the slip surface is best remedied by changing the inclination of the slip surface so that it corresponds more closely to the inclination of the shear plane for passive earth pressures.
- The problem of inappropriate stresses at the toe of a slope may also be remedied by (1) estimating shear strength directly based on Rankine passive earth pressure theory, (2) using the Ordinary Method of Slices, or (3) changing the interslice force inclination.

OTHER DETAILS

Additional details that affect the results of slope stability calculations include the number of slices into which the soil mass is divided, and the tolerances used to define convergence in the iterative procedures for calculating factors of safety.

Iteration Tolerances and Convergence

All procedures except the infinite slope and the Ordinary Method of Slices procedures require an iterative (trial-and-error) process to calculate the factor of safety for a given slip surface. The iterative process requires one or more convergence criteria to define when iterations can be stopped and a sufficiently accurate solution for the factor of safety has been found. Convergence criteria can be specified in terms of maximum allowable changes in the calculated values of factor of safety on successive iterations, or limits on the magnitude of allowable force and moment imbalance, or on a combination of these criteria. The tolerances for convergence may be embedded in the software or they may be included as part of the input data. In either case, awareness of the criteria and the possible consequences is important.

When a search is being conducted to locate a critical slip surface the convergence criteria for the factor of safety must be smaller than the changes in factor of safety between adjacent slip surfaces analyzed in the search process. If the convergence criteria produce factors of safety that are less precise than the changes that occur when the slip surface is moved small distances, the search can result in false minimums for the factor of safety. For typical slopes and search schemes, it is appropriate for convergence criteria to produce factors of safety that are precise to 0.0001.

Convergence criteria for force and moment imbalances need to be chosen carefully, depending on the problem being solved. Criteria that require force and moment equilibrium to be satisfied to within 100 lb and 100 ft-lb usually work well. However, for very shallow slides these limits should be reduced. Slip surfaces for very shallow slides that approximate infinite slope failures sometimes involve less than 100 lb of total weight and a precision of 100 lb force imbalance is not adequate; a 100-lb force imbalance could cause twofold variations in the computed factors of safety. In contrast to very shallow slides where very small imbalance tolerances are required, massive slides may require larger than usual equilibrium tolerances. For example, Gucwa and Kehle (1978) describe slope stability analyses for a massive landslide in the Bearpaw Mountains of Montana. The lateral extent of the slide from the crest to the toe of the slide mass was in excess of 10 km, and the depth of the slip surface may have approached 1 km. For analyses of the slide it was necessary to use as acceptable force imbalances values that were several orders of magnitude greater than the usual 100 lb.

An additional level of iteration is required to calculate the factor of safety when the Mohr failure envelope is curved rather than linear. In such cases the shear strength parameters (c, c' and ϕ, ϕ') vary with the normal stress, but the normal stress can only be determined once the shear strength parameters are known. Only the infinite slope and Ordinary Method of Slices procedures permit the normal stress to be calculated independently of the shear strength parameters. Any of the other limit equilibrium procedures requires trial-and-error solutions when curved (nonlinear) Mohr failure envelopes are used. In the trial-and-error procedures, the shear strength is estimated, the factor of safety and normal stresses are calculated, new strengths are calculated, and the process is repeated until the assumed and calculated shear strengths are sufficiently alike. Some tolerance must either be built into or entered as input data into any computer program that performs such calculations. If the tolerances are too large, they can result in inaccurate solutions or false local minima for the factor of safety.

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Number of Slices

All of the procedures of slices involve subdividing the soil above the slip surface into slices. The number of slices used depends on several factors, including the complexity of the soil profile, whether the calculations are performed by hand or by a computer program, and accuracy requirements.

Required slices. Most slopes have several points where it is either convenient or necessary to place slice boundaries. For example, boundaries are usually placed wherever there is a break in the slope profile (points *b* and *f* in Figure 14.18), wherever a stratum outcrops on the slope surface (point *c* in Figure 14.18), or wherever the slip surface crosses a boundary between two different strata (points *a*, *d* and *e* in Figure 14.18). By placing slice boundaries at such points, the base of each slice will be in only one material, and the soil strata will vary continuously and smoothly across each slice. Slice boundaries may also be placed where there is a change in the slope of a piezometric surface, where the slip surface crosses either the water table or piezometric line, and where there is an abrupt change in distributed load on the surface of the slope.

For circular slip surfaces, once the required slice boundaries are established, additional slices are added to achieve a more precise approximation of the curved slip surface by straight-line segments (the bases of the slices). Additional slices may also be appropriate to account better for variations in shear strength or pore water pressure along a slip surface. Also, the number of slices used depends on whether the calculations are being performed by hand or by computer.

Hand calculations. When calculations are done by hand, the achievable level of accuracy is on the order of 1%. The number of slices needed to be consistent with this accuracy is usually no more than 8 to 12. Once the required number of slices is set, a few additional slices may be added to make the slices more uniform in size. For complex geometries with a variety of piezometric surfaces and distributed loads, as many as 30 to 50 slices are sometimes required to capture all of the details in the cross section. This number (30 to 50), however, is exceptional. It would be unusual to use hand calculations for such a problem except to verify a computer solution.

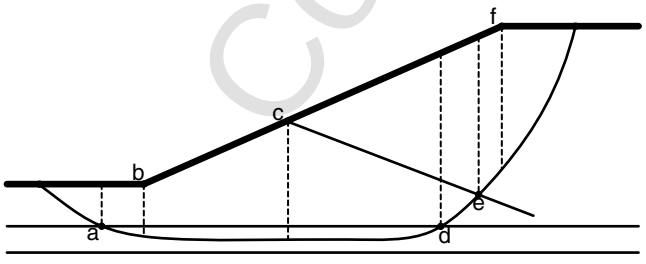


Figure 14.18 Locations where slice boundaries are required.

Computer calculations. When calculations are performed using a computer, rather than by hand, many more slices are used. Very little time is required to perform the calculations with a computer, even when 50 or more slices are used.

For circles, the effect of the number of slices on the factor of safety is best related to the angle, θ_s, subtended by radii drawn to each side of the base of each slice (Figure 14.19). It has been found that the accuracy of a solution is related more to the angle, θ_s, than to the number of slices (Wright, 1969). To illustrate the effect of the subtended angle and number of slices, slope stability computations were performed for the two slopes shown in Figure 14.20. For the slope shown in the lower part of Figure 14.20, two different representations of undrained shear strength were used for the foundation, and separate computations were performed for each. The subtended angle used to generate slices was varied from 1 to 40°. Results for the three different slope shear strength combinations are summarized in Table 14.1. Several conclusions can be drawn from these results:

- Factors of safety are essentially the same (maximum difference 0.1%) using 1° and 3° as the maximum subtended angle for subdivision into slices.
- The factor of safety tends to increase very slightly (less than 1%) as the subtended angle is increased from 3° to 15°.
- For very large subtended angles (more than 15°) the factor of safety may either increase or decrease, depending on the particular slope geometry and variation in soil properties.

It is very likely that the factor of safety could vary differently with the number of slices, depending on the particular algorithms employed to compute the slide weights, length of slip surface, and the center of gravity and moment arms for slices. However, it seems unlikely that there will be any significant differences or errors if the slices are subdivided using a subtended angle of 3° or less.

Based on the discussion above, the most appropriate way to set the number of slices is to set an upper limit for the subtended angle, θ_s. A value of 3° works well for this purpose, and 3° typically results in from 30 to 60 slices, assuming total subtended angles for the slip surface of 90 to 180°. For deep slip surfaces with large total subtended angles, 50 or more slices may be re-

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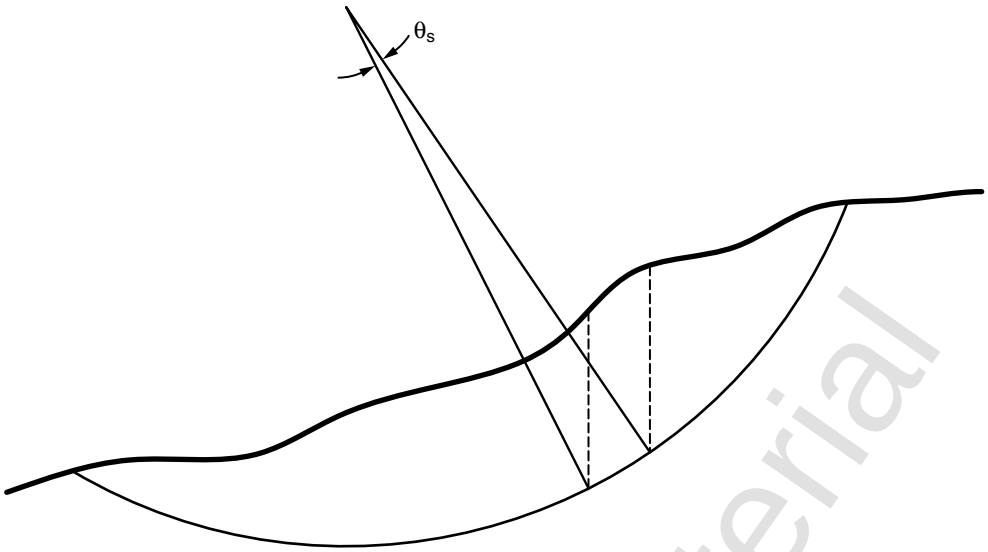
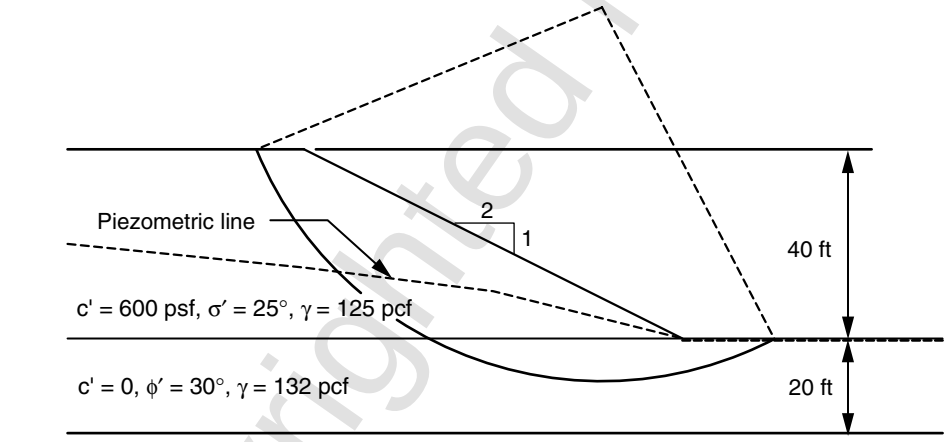
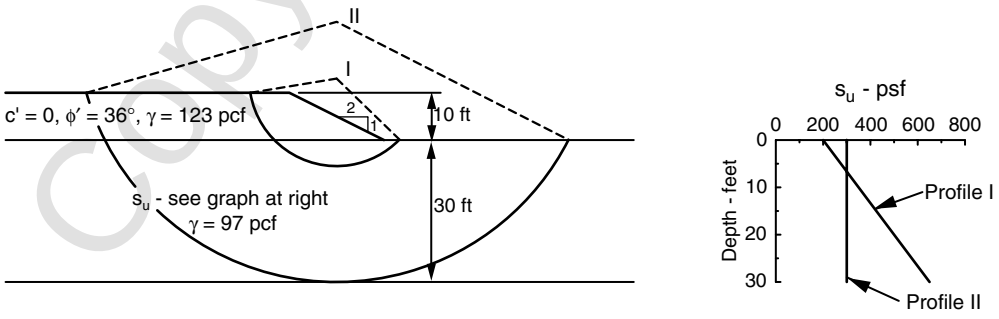


Figure 14.19 Subtended angle, θ_s, used to characterize the size of slices.



(*a*)



(*b*)

Figure 14.20 Slopes used to illustrate the effect of subdivision into slices: (*a*) example slope 1; (*b*) example slope 2.

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Table 14.1 Variations in Factors of Safety with the Subtended Angle Used for Subdivision of the Soil Mass into Slices[a]

Subtended angle (deg)	Factor of safety		
	Example slope 1	Example slope 2–strength profile I	Example slope 2–strength profile II
1	1.528	1.276	1.328
3	1.529	1.276	1.328
	(0.1)	(0.0)	(0.0)
5	1.530	1.277	1.329
	(0.1)	(0.1)	(0.1)
10	1.535	1.279	1.332
	(0.5)	(0.2)	(0.3)
20	1.542	1.291	1.331
	(0.9)	(1.2)	(0.2)
30	1.542	1.323	1.314
	(0.9)	(3.7)	(−1.1)
40	1.542	1.323	1.295
	(0.9)	(3.7)	(−2.5)

[a]Numbers in parentheses represent the percent increase (+) or decrease (−) in factor of safety compared to the factor of safety when the soil mass is subdivided into slices using a subtended angle of 1°.

quired for a subtended angle of 3°. On the other hand, for very shallow slip surfaces that approximate the infinite slope mechanism, very few slices are required to achieve the same degree of accuracy. The number of slices will also depend on the complexity of the slope geometry and the number of interslice boundaries required.

In some cases where circles are very shallow, a maximum subtended angle of 3° may produce only one slice. This causes the base of the slice (slip surface) to coincide with the surface of the slope, and the slice has zero area. When this occurs, an arbitrary number of slices is used. The number of slices is not important—as few as two or three slices will suffice. In these cases it is more appropriate to subdivide the slip surface into slices using a prescribed arc length rather than a prescribed subtended angle. The arc length can be chosen so that any slip surface that is of sufficient length to be of interest will be divided into 5 to 10 slices. For example, if slip surfaces with lengths of 10 ft or more are judged to be of interest, a maximum arc length of 2 ft could be used for subdividing into slices.

The number of slices used to represent noncircular slip surfaces has an effect on the computed factor of safety, but the criterion for the number of slices cannot be represented by a subtended angle. Instead, a minimum number of slices is usually chosen; usually, 30 or more slices works well. Similarly to circles, the best results are obtained by subdividing the soil mass such that the lengths of the base of slices, Δl, are approximately equal, rather than using slices with equal widths, Δx. Use of a constant base length along the slip surface produces more slices at the more steeply inclined ends of the slip surface and fewer where the slip surface is flatter (Figure 14.21).

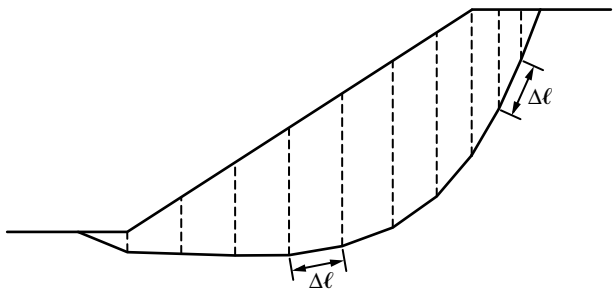


Figure 14.21 Subdivision for slices with a noncircular slip surface so that the bases of slices are of approximately constant length.

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Recapitulation

- Convergence criteria that are too coarse can result in false minima and an incorrect location for the critical slip surface.
- Convergence criteria for force and moment imbalance should be scaled to the size of the slope. Tolerances of 100 lb/ft (0.1 kN/m) and 100 ft-lb/ft (0.1 kNm/m) are suitable for most slopes.
- The number of slices does not have a large effect on the computed factor of safety, provided that details of the slope and subsurface stratigraphy are represented.
- For hand calculations only a small number of slices (6 to 12) is required to be consistent with the accuracy achievable by hand calculations.
- For computer solutions with circular slip surfaces, the number of slices is usually chosen by selecting a maximum subtended angle, θ_s, of 3° per slice.
- For computer solutions with noncircular slip surfaces, 30 or more slices are used. Slices are subdivided to produce approximately equal lengths for the base of the slices along the slip surface.

VERIFICATION OF CALCULATIONS

Any set of slope stability calculations should be checked by some independent means. Numerous examples of how analyses can be checked have already been presented in Chapter 7. Also, Duncan (1992) summarized several ways in which the results of slope stability calculations can be checked, including:

1. Experience (what has happened in the past and what is reasonable)
2. By performing extra analyses to confirm the method used by comparison with known results
3. By performing extra analyses to be sure that changes in input causes changes in results that are reasonable
4. By comparing key results with computations performed using another computer program, slope stability charts, spreadsheet, or detailed hand calculations

Many slope stability calculations are performed with a computer program that uses a complete equilibrium procedure of slices. Most of these complete equilibrium procedures (Spencer, Morgenstern and Price, Chen and Morgenstern, etc.) are too complex for calculation by hand. In this case suitable manual checks of the calculations can be made using force equilibrium procedures and assuming that the interslice forces are inclined at the angle(s) obtained from the computer solution. Suitable spreadsheets for this purpose and example calculations are presented in Chapter 7.

Published benchmark problems also provide a useful way for checking the validity of computer codes, and although benchmarks do not verify the solution for the particular problem of interest, they can lend confidence that the computer software is working properly and that the user understands the input data. Several compilations of benchmark problems have been developed and published for this purpose (e.g., Donald and Giam, 1989; Edris and Wright, 1987).

In addition to benchmark problems there are several ways that simple problems can be developed to verify that computer codes are working properly. Most of these simple problems are based on the fact that it is possible to model the same problem in more than one way. Examples of several simple test problems are listed below and illustrated in Figures 14.22 and 14.23:

1. Computation of the factor of safety for a submerged slope under drained conditions using:
   a. Total unit weights, external loads representing the water pressures, and internal pore water pressures
   b. Submerged unit weights with no pore water pressure or external water loads

   This is illustrated in Figure 14.22*a*. Both approaches should give the same factor of safety.[3]
2. Computation of the factor of safety with the same slope facing to the left and to the right (Figure 14.22*b*). The factor of safety should not depend on the direction that the slope faces.
3. Computation of the factor of safety for a partially or fully submerged slope, treating the water as:
   a. An externally applied pressure on the surface of the slope
   b. A ''soil'' with no strength ($c = 0$, $\phi = 0$) and having the unit weight of water.

   This is illustrated in Figure 14.22*c*.
4. Computation of the factor of safety for a slope with very long internal reinforcement (geogrid, tieback) applying the reinforcement loads as:
   a. External loads on the slope
   b. As internal loads at the slip surface

[3] See also the discussion in Chapter 6 of water pressures and how they are handled. Large differences between the two ways of representing water pressures may occur if force equilibrium procedures are used. See also Example 2 in Chapter 7.

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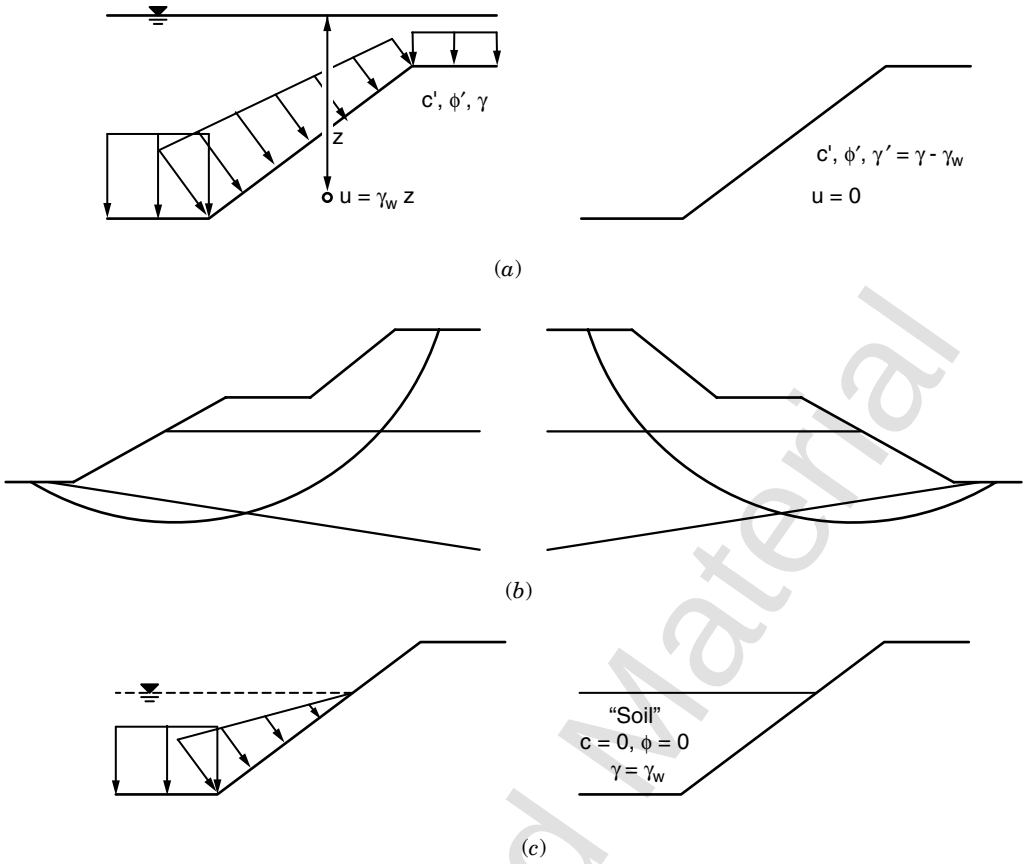


Figure 14.22 Equivalent representations for selected slope problems: (*a*) simple submerged slope, no flow; (*b*) left- and right-facing slope; (*c*) partially submerged slope.

This is illustrated in Figure 14.23*a*. Although intuitively the location where the force is applied might be expected to have an effect, the location does not have a large effect on the computed factor of safety, provided that the slip surface does not pass behind the reinforcement and the force does not vary along the length of the reinforcement.

5. Computation of the bearing capacity of a uniformly loaded strip footing on a horizontal, infinitely deep, purely cohesive foundation (Figure 14.23*b*). For circular slip surfaces and a bearing pressure equal to 5.53 times the shear strength (c) of the soil, a factor of safety of unity should be calculated.
6. Computation of the seismic stability of a slope using:
   a. A seismic coefficient, k
   b. No seismic coefficient, but the slope is steepened by rotating the entire slope geometry through an angle, θ, where the tangent of θ is the seismic coefficient (i.e., $k = \tan\theta$) *and* the unit weight is increased by multiplying the actual unit weight by $\sqrt{1 + k^2}$

   This is illustrated in Figure 14.23*c*. In both solutions the magnitude and direction of the forces will be the same and should produce the same factor of safety.

Additional test problems can also be created using slope stability charts like the ones presented in the Appendix.

THREE-DIMENSIONAL EFFECTS

All the analysis procedures discussed in Chapter 6 assume that the slope is infinitely long in the direction perpendicular to the plane of interest; failure is assumed to occur simultaneously along the entire length of the slope. A two-dimensional (plane strain) cross section is examined, and equilibrium is considered in

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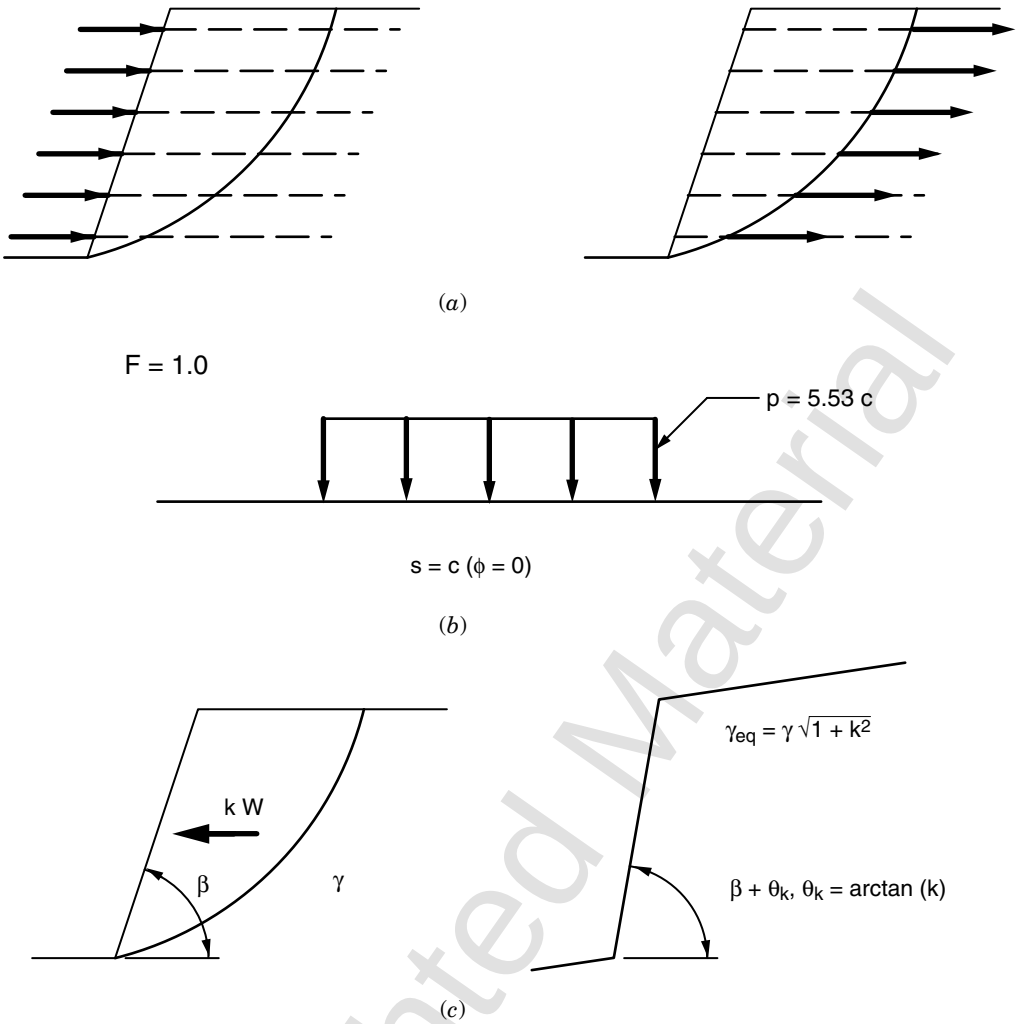


Figure 14.23 Equivalent representations for selected slope problems: (*a*) reinforced slope; (*b*) bearing capacity on saturated clay; (*c*) pseudostatic analyses.

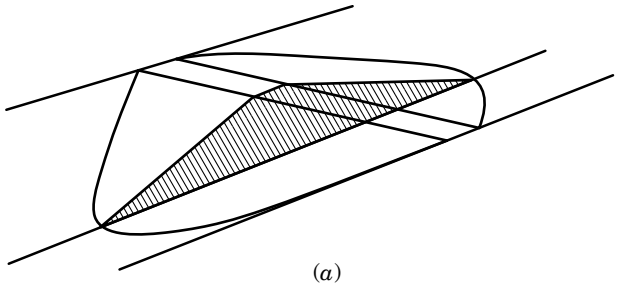
just two directions. On the other hand, most slope failures are finite, and most failure surfaces are three-dimensional, often being bowl-shaped.

Many three-dimensional failures occur because soil properties and pore water pressures vary along the length of the slope (i.e., subsurface conditions and soil properties are not uniform). Generally, there are not enough data to describe the variation in properties along the length of a slope in sufficient detail to perform more rigorous analyses that account for spatial variations.

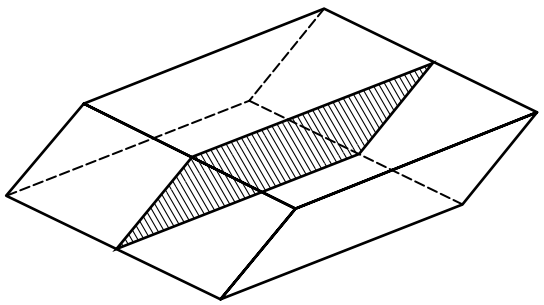
Three-dimensional slope failures may also occur due to the three-dimensional geometry of the slope. Several analysis methods have been developed to address the effects of three-dimensional geometry. Comparisons can be and have been made between results of two- and three-dimensional analyses. In comparing results of three-dimensional slope stability analysis with those from two-dimensional analyses, it is important that the cross section used for the two-dimensional analyses be stipulated. Two-dimensional analyses are generally performed either for the maximum cross section or for the cross section that gives the lowest factor of safety. The maximum cross section is the cross section where the slope is highest or where the maximum amount of soil is involved in potential sliding. The maximum cross section for a dam is usually near the center of the valley (Figure 14.24*a*); the maximum cross section for a waste fill may be perpendicular to an exposed face of the fill, approximately midway between two opposing, lateral side slopes (Figure 14.24*b*). However, the maximum cross section does not always produce

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(*a*)



(*b*)

Figure 14.24 Typical ''maximum'' cross sections used for the two-dimensional analysis of three-dimensional geometries: (*a*) earth dam; (*b*) waste fill.

the minimum factor of safety. This was clearly demonstrated by Seed et al. (1990) for the analyses of a slide that occurred at the Kettleman Hills landfill. Factors of safety calculated for the maximum sections were 1.10 to 1.35, while factors of safety for smaller sections near the side slopes were as low as 0.85. Conclusions drawn regarding differences between two- and three-dimensional analyses will thus depend on whether the two-dimensional analyses are performed for the maximum cross section or for the cross section with the minimum factor of safety. Very different conclusions may be drawn regarding differences depending on which two-dimensional cross section is used for the analyses.

The two-dimensional factor of safety for the most critical (lowest factor of safety) section will always be lower than the three-dimensional factor of safety, provided that comparable limit equilibrium approaches are used (i.e., similar assumptions and rigor are used to satisfy static equilibrium).

Most of the general-purpose three-dimensional slope stability analysis procedures are based on a method of columns. The methods of columns are the three-dimensional equivalent of the two-dimensional procedures of slices. In the method of columns the soil mass is subdivided into a number of vertical columns, each with an approximately square cross section in plan view. A considerable number of assumptions must be made to achieve a statically determinate solution with the method of columns. Several procedures employ simplifying approaches comparable to the Ordinary Method of Slices, rather than fully satisfying the six equations[4] of static equilibrium. The effects of these assumptions may be as large as the three-dimensional effects themselves. Thus, a considerable amount of uncertainty exists in the results from many of the three-dimensional procedures, and the procedures should be used cautiously, especially when they are being used as a basis for acceptance when two-dimensional analyses might indicate unacceptably low factors of safety.

Hutchinson and Sharma (1985) and Leshchinsky and Baker (1986) pointed out that for cohesionless soils two- and three-dimensional analyses should give the same factor of safety because the critical slip surface is a plane coincident with the surface of the slope. Unless there is significant cohesion or the geometry and soil strengths are such that the failure surface is deep, the difference between two- and three-dimensional analyses is likely to be small.

Azzouz et al. (1981) and Leshchinsky and Huang (1992) both noted that when shear strengths are back-calculated using two-dimensional analyses for three-dimensional conditions, the shear strengths will be too high. This error is compensated, however, if the shear strengths are subsequently used in two-dimensional analyses of similar conditions.

Most slope stability analyses neglect three-dimensional effects without significant consequence. However, there are two practical instances where two-dimensional analyses might not be adequate, and three-dimensional analyses would be warranted:

1. When strengths are back-calculated from three-dimensional failures and the bias in the back-calculated strengths would not be compensated in subsequent analyses
2. When, due to the slope geometry, there may be a significant benefit of the three-dimensional effects that will improve stability

In these cases, three-dimensional analyses may be warranted.

[4] Three equations for force equilibrium and three equations for equilibrium of moments about three axes.

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